Mid-Term II

Time: 45 min

Physical and Astronomical Constants

CODATA Internationally recommended values of the Fundamental Physical Constants.
The most recent values can be found at `http://physics.nist.gov/cuu/Constants`.

Solar mass	$M_\odot$	=	$1.989 \cdot 10^{33}$ gm
Solar radius	$R_\odot$	=	$6.955 \cdot 10^{10}$ cm
Solar effective temperature	$T_{\mathrm{eff},\odot}$	=	5780 K
Solar surface gravity	$g_{\mathrm{s},\odot}$	=	$2.744 \cdot 10^{4}$ cm/sec^2
Solar luminosity	$L_\odot$	=	$3.846 \cdot 10^{33}$ erg/sec
Solar absolute bol. mag.	$M_{b,\odot}$	=	+4.77
Velocity of light in vacuo	c	=	$2.99792458 \cdot 10^{10}$ cm/sec
Constant of gravitation	G	=	$6.6742 \cdot 10^{-8}$ cm^3/(gm s^2)
Boltzmann constant	k	=	$1.3807 \cdot 10^{-16}$ erg/K
Avogadro's number	N_0	=	$6.022 \cdot 10^{23}$ mole^{-1}
Atomic mass unit	1 AMU	=	$1/N_0 = H$
		=	$1.66054 \cdot 10^{-24}$ gm = 931.5 MeV
Gas constant	$\mathcal{R}$	=	$8.314 \cdot 10^{7}$ erg/K/mole
Planck's constant	h	=	$6.626 \cdot 10^{-27}$ erg sec
	$\hbar = h/2\pi$	=	$1.0546 \cdot 10^{-27}$ erg sec
Electronic charge	e	=	$4.803 \cdot 10^{-10}$ e.s.u.
		=	$1.602 \cdot 10^{-19}$ C
Fine structure constant	$e^2/\hbar c$	=	1/137.036
Stefan-Boltzmann constant	σ	=	$5.670 \cdot 10^{-5}$ erg/(cm^2 K^4 sec)
Radiation pressure constant	$a = 4\sigma/c$	=	$7.566 \cdot 10^{-15}$ erg/(cm^3 K^4)
Electron rest mass	m_e	=	$9.109 \cdot 10^{-28}$ gm = 0.5110 MeV
Mass ratio proton/electron	m_p/m_e	=	1836.2
Mass of hydrogen atom	H^1	=	$1.6734 \cdot 10^{-24}$ gm
		=	1.0081 AMU
Classical electron radius	e^2/m_ec^2	=	$2.818 \cdot 10^{-13}$ cm
Compton wavelength of electron	$\lambda_C = \hbar/m_ec$	=	$3.8616 \cdot 10^{-11}$ cm
Thomson scattering cross section	σ_0	=	$(8\pi/3)\left(e^2/m_ec^2\right)^2$
		=	$0.6652 \cdot 10^{-24}$ cm^2
Electron volt	1 eV	=	$1.602 \cdot 10^{-12}$ erg = 11604 K

Please use cgs units for calculations and numerical values.

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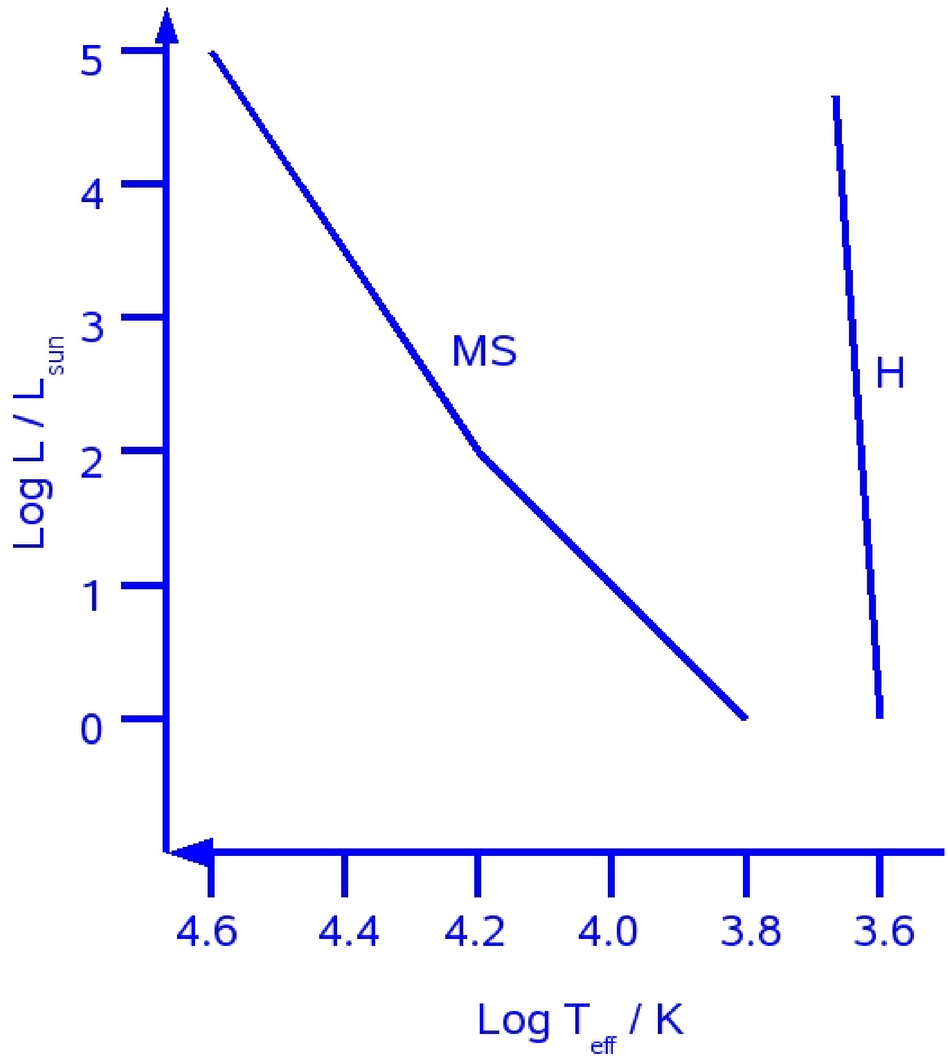
1. STARS IN THE HERTZSPRUNG-RUSSELL DIAGRAM

   The Hertzsprung-Russell Diagram (HRD) connects observational properties of stars

   (a) **Draw the appropriate axes for a HRD with axis ranges appropriate for stars like to massive stars. Label the axes.**

   Score: **2**



   (b) **In your diagram above, indicate the location of the main sequence and label it "MS".** Score: **2**

   (c) **In your diagram above, indicate the location of the Hayashi Line and label it "H".** Score: **2**

   (d) **What is the mode of energy transport for stars on the Hayashi Line? That is, what is specific about their structure?**

   Score: **2**

   These stars are fully convective.

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2. Stellar Populations

(a) **Which lives longer: a $2\,\mathsf{M}_\odot$ star or a $10\,\mathsf{M}_\odot$ star?**

Score: **2**

A $2\,\mathsf{M}_\odot$ star lives longer than a $10\,\mathsf{M}_\odot$ star

(b) **What does the initial mass function (IMF) describe, what does the birth function describe?**

Score: **4**

- The number of stars in a mass bin $[M, M+\mathsf{d}M]$ is called the **birth function** $\Phi(M)$:

$$\mathsf{d}N = \Phi(M)\mathsf{d}M$$

- The mass of stars in a mass bin is then given by weighing by mass M, defining the **initial mass function (IMF)** $\xi(M)$:

$$\xi(M) = M\mathsf{d}N/\mathsf{d}M$$

Salpeter (1955) found observationally a power law for Φ, ξ:

$$\Phi(M) \propto M^{-2.35}\,, \quad \xi(M) \propto M^{-1.35}$$

(c) **For the common Salpeter IMF, are there more $2\,\mathsf{M}_\odot$ stars or more $10\,\mathsf{M}_\odot$ stars made?**

Score: **2**

There are more $2\,\mathsf{M}_\odot$ stars made.

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3. Nuclear Burning

(a) **What are the two main products (isotopes) of helium burning?**

Score: **2**

$^{12}\mathsf{C}$, $^{16}\mathsf{O}$

(b) **What happens to $^{14}\mathsf{N}$ during helium burning? Write down the reaction path.**

Score: **4**

First $^{14}\mathsf{N}(\alpha,\gamma)^{18}\mathsf{F}(\mathsf{e}^{+}\nu_{\mathsf{e}})^{18}\mathsf{O}(\alpha,\gamma)^{22}\mathsf{Ne}$,
later it is source of the neutrons for the weak *s*-process, $^{22}\mathsf{Ne}(\alpha,\mathsf{n})^{25}\mathsf{Mg}$

(c) **In what stars and what stage does silicon burning occur?**
What element(s) does it make?

Score: **4**

It occurs at the end of the evolution of massive stars. It makes iron and iron-group elements.

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4. Nucleosynthesis

   (a) **Name the three major nucleosynthesis mechanisms (processes) contributing to the production of heavy elements beyond the iron group. In what stars and during what stage of evolution do they occur?**

Score: **8**

- s-process:
  intermediate-mass stars during the AGB phase, strong component of the s-process; massive stars at the end of helium burning, and during helium and carbon shell burning.
- r-process: during supernova explosion of massive stars
- p-process: during supernova explosion of massive stars

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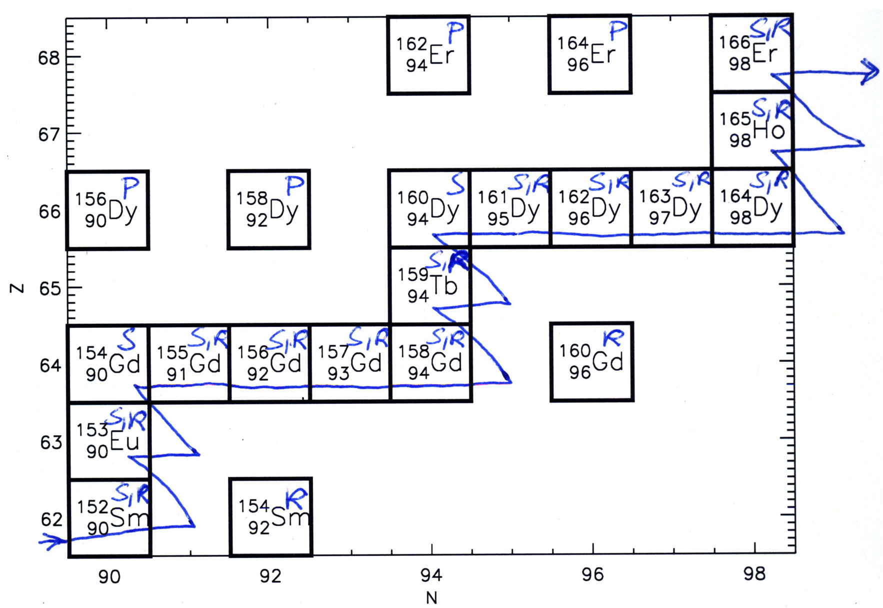
(b) The figure below show stable isotopes in the chart of nuclei.
Indicate in the figure which nuclei are made by which process.

Score: **6**



In the figure above I indicate

- S: *s*-process elements
- R: *r*-process elements
- P: *p*-process elements

NOTE: From the limited extend of the figure itself it may not be possible to tell the right answers for ^{152}Sm, ^{153}Eu, ^{165}Ho, and ^{166}Er.

(c) **One of the processes follows a distinct path. Indicate that path in the figure above. Assume that all weak decays are fast compared to captures.**

Score: **4**

See path drawn above.
NOTE: From the limited extend of the figure itself it may not be possible to tell the right answers for ^{152}Sm, ^{153}Eu, ^{154}Gd, ^{165}Ho, and ^{166}Er.

5. MASS LIMITS AND STELLAR EVOLUTION

(a) **For a white dwarf made of non-relativistic degenerate gas, how does its radius change with increasing mass of the white dwarf?**

Score: **2**

$$R \propto M^{-1/3}$$

The radius decreases with increasing radius.

(b) There is a maximum mass for white dwarf stars.

How is this limit called and what is the reason for this limit. Explain the physical mechanism. For a star made of pure helium, give a rough numerical value (2-3 significant digits)

Score: **6**

This Limit is called the Chandrasekhar Mass. It occurs because as the mass of the white dwarf stars increases, the radius decreases and hence the density increases, until the relativistic degenerate equation of state has to be used. For these stars, a polytrope of index $n = 3$ has to be used, with a unique solution for mass.

$$M_{\mathsf{Ch}} = \frac{M_3}{4\pi}\left(\frac{3}{2}\right)^{1/2}\left(\frac{hc}{G\mathsf{u}^{4/3}}\right)^{3/2}\mu_{\mathsf{e}}{}^{-2} = (5.836\,\mathsf{M}_\odot)\mu_{\mathsf{e}}{}^{-2} = 1.459\,\mathsf{M}_\odot\left(\frac{\mu_{\mathsf{e}}}{2}\right)^{-2}$$

For pure helium we have $\mu_{\mathsf{e}} = 2$ and hence $M_{\mathsf{Ch}} = 1.459\,\mathsf{M}_\odot$

(c) **What is the Schönberg-Chandrasekhar Limit, and for what stars (initial stellar masses) is it relevant?**

Score: **4**

The Schönberg-Chandrasekhar Limit gives a minimum mass, more precisely what mass fraction of the total mass of the core after end of central hydrogen burning, the core has to exceed so it can contract and ignite helium burning. It is relevant for low-mass stars below about $2\,\mathsf{M}_\odot$.