Neutrinos & Origin of Elements

PHY 8850

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Neutrinos & Origin of Elements, Spring 2009

Agenda

1. Introduction
   - Basic Assumptions About Stars
   - The Sun
   - Stellar Equation of State
2. Burning in Stars

Overview

1 Introduction
- Basic Assumptions About Stars
- The Sun
- Stellar Equation of State

2 Burning in Stars

What are Stars?

Stars

1. are bound by self-gravity
2. radiate energy supplied by an internal source

Usually stars have a nuclear energy source

Energy Sources

What energy sources are conceivable?

- gravitational binding energy
  - contraction
  - gravitational settling
- nuclear energy / burning
- chemical energy / burning
- heat capacity (just cooling down)
- pulsation energy dissipation
- rotational energy dissipation

What are Stars? (continued)

- Stars usually live and shine steadily for a long time
- From (1) follows that stars usually are spherical unless they rotate strongly
- Planets mostly shine by reflection of sun light
- Because stars radiate - lose energy - energy conservation requires that they must evolve; they burn nuclear fuel
- “Death” of stars by disruption or running out of fuel; often a combination of both (“compact” remnant formation - white dwarf, neutron stars, black hole, in the latter two cases a powerful “supernova” may occur in the process)

What are Stars? (continued)

- Star formation is very complicated
- We will follow stars from the early time when they fulfill conditions (1) and (2)
- Galaxies are large systems of stars, some $10^6 \ldots 10^{12}$
- Clusters of galaxies can contain some 100,000 galaxies

The Sun

- Luminosity
  $L_\odot = 3.84 \times 10^{33}$ erg/s $= 3.84 \times 10^{26}$ J/s
- Mass
  $M_\odot = 1.98 \times 10^{33}$ g $= 1.98 \times 10^{30}$ kg
- Radius
  $R_\odot = 6.98 \times 10^{10}$ cm $= 6.98 \times 10^{5}$ km

Gravitational Binding Energy

Binding energy can be approximated by

$$E = \frac{\mathrm{G} M^2}{2R} \quad , \qquad G = 6.67259 \times 10^{-8} \frac{\mathrm{cm}^3}{\mathrm{g\,s}}$$

The lifetime of the star is then defined by how long it takes to radiate away that energy, hence dividing by luminosity

$$\tau_{\mathrm{KH}} = \frac{E}{L} = \frac{\mathrm{G} M^2}{2RL}$$

This is called the *Kelvin-Helmholtz time-scale*.
It tells how long a star takes to radiate away it gravitational binding energy. This is also the time-scale for stars to get in *gravo-thermal* equilibrium.

Gravitational Binding Energy

For the Sun we obtain

$$\tau_{\mathrm{KH},\odot} = \frac{\mathrm{G}M_\odot^2}{2R_\odot L_\odot}$$

$$\tau_{\mathrm{KH},\odot} = 4.9 \times 10^{14}\,\mathrm{s} = 15.6 \times 10^6\,\mathrm{yr}$$

Basic Assumptions

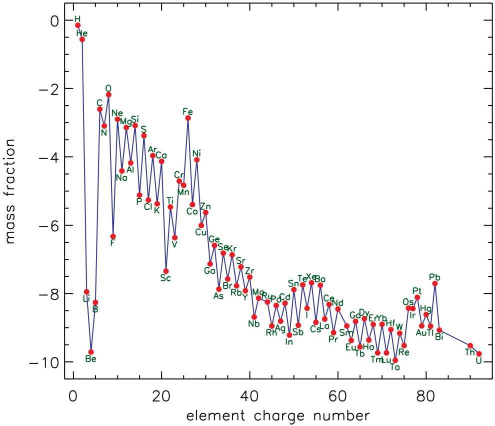
- stars evolve *in isolation*
  distances between stars are large compared to their radii
- spherical symmetry
  sun rotates once in 27 days, $\omega \approx 2.5 \times 10^{-6}$/s

$$\frac{M\omega^2 R^2}{GM^2/R} = \frac{\omega^2 R^3}{GM} \approx 2 \times 10^{-5}$$

- only small variation in (initial) composition of stars
  sun: $X = 0.70,\ Y = 0.28,\ Z = 0.02,\ X + Y + Z = 1$
- small magnetic fields - even for $B \sim 0.1\,\mathrm{T}$:

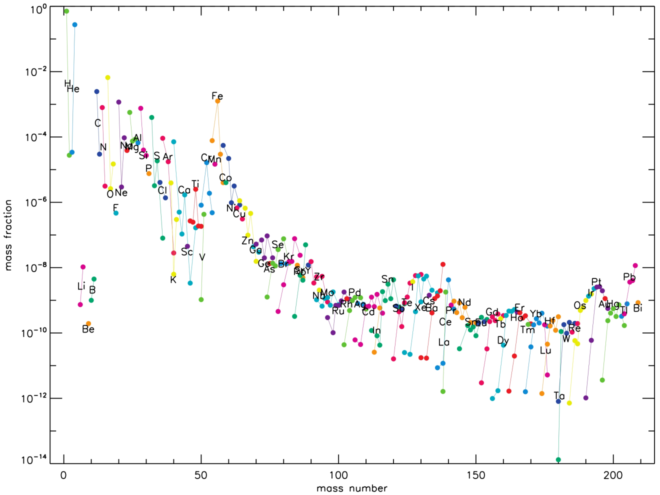
$$\frac{B^2/\mu_0}{GM^2/R^4} = \frac{B^2 R^4}{\mu_0 GM^2} \sim 10^{-11}$$

The Solar Abundance Pattern (Elements)

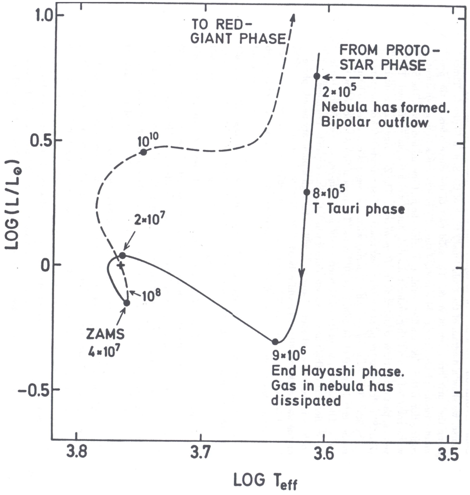


The solar abundance pattern, by mass fraction of elements.

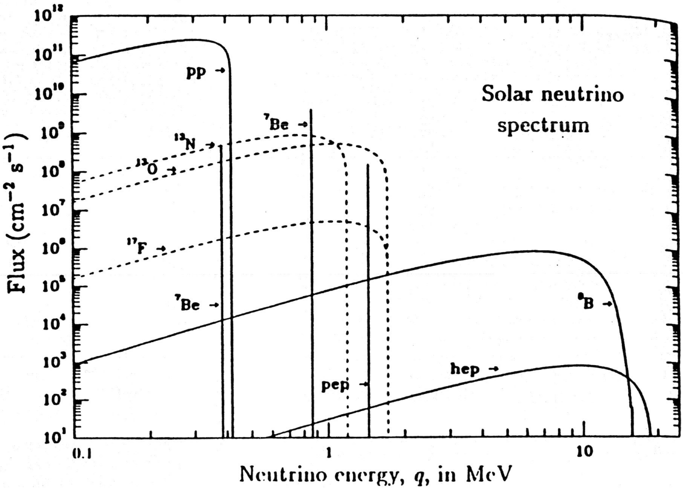
The Solar Abundance Pattern (Isotopes)



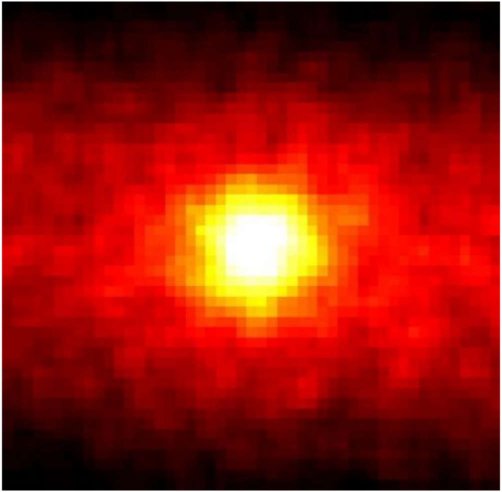
Evolution of the Sun in the HRD



The Solar Neutrino Spectrum



The Sun as Seen in Neutrinos



Stellar Structure Equations

stationary terms   time-dependent terms

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \tag{1}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} \tag{2}$$

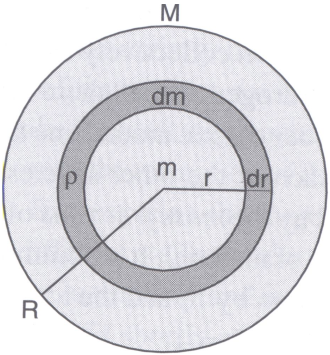
$$\frac{\partial F}{\partial m} = \varepsilon_{\mathrm{nuc}} - \varepsilon_\nu - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} \tag{3}$$

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla \left[1 + \frac{r^2}{Gm}\frac{\partial^2 r}{\partial t^2}\right] \tag{4}$$

$$\frac{\partial X_i}{\partial t} = f_i\left(\rho, T, \mathbf{X}\right) \tag{5}$$

where $\mathbf{X} = \{X_1, X_2, \ldots, X_i, \ldots\}$.

Relation between mass and radius



- integral formulation:

$$m(r) = \int_0^r 4\pi r^2 \rho(r)\,\mathrm{d}r$$

- differential formulation

$$\mathrm{d}m = 4\pi\rho r^2 \mathrm{d}r$$

Stellar Gas

- stellar “gas” composed ions, electron, and radiation
- radiation regarded as “photon gas” with quanta caring $h\nu$ energy and $h\nu/c$ momentum
- photon gas described by Planck spectrum
- ion/electron gas described by Maxwellian velocity distribution
- at high density and low temperature electron gas follows *degenerate* equation of state (Fermi statistics)
- at even lower T and higher ρ ions (nucleons) can be degenerate (e.g., neutron stars)

Summary

- Equation of Motion

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = -\frac{Gm}{r^2} - 4\pi r^2 \frac{\partial P}{\partial m}$$

- hydrostatic equilibrium

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}$$

- change of composition

$$\frac{\partial X_i}{\partial t} = f_i\left(\rho, T, \mathbf{X}\right) = f_{i,\mathrm{nuc}}\left(\rho, T, \mathbf{X}\right) + f_{i,\mathrm{mix}}\left(\rho, T, \mathbf{X}\right)$$

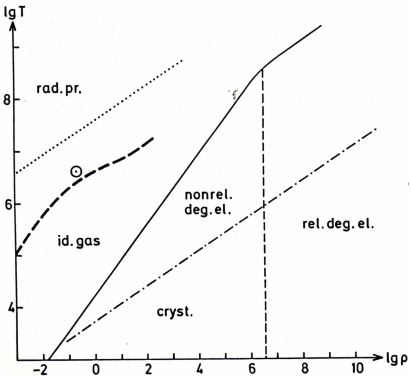
- nuclear reactions

$$\frac{\partial}{\partial t} Y_i = \sum_{\substack{\alpha_1, \alpha_2, \ldots \\ \beta_1, \beta_2, \ldots}} \lambda_{\alpha_1 \mathbf{1} + \alpha_2 \mathbf{2} + \ldots \rightarrow \beta_1 \mathbf{1} + \beta_2 \mathbf{2} + \ldots} \frac{\beta_i - \alpha_i}{\alpha_1 ! \alpha_2 ! \ldots} Y_1^{\alpha_1} Y_2^{\alpha_2} \ldots$$

Summary of Pressure Contributions

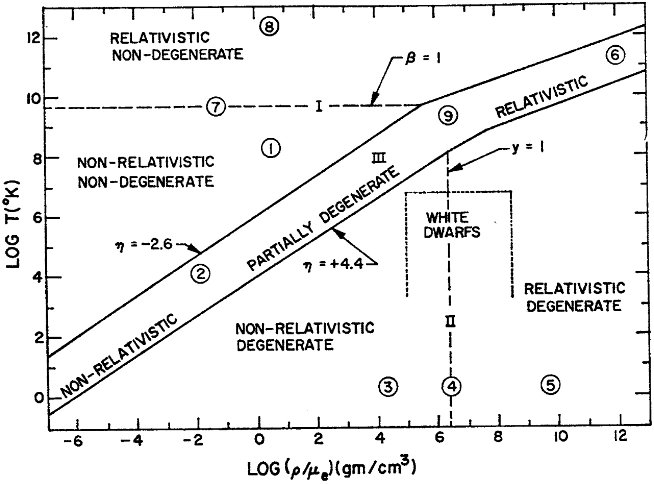
- Pressure integral $P = \frac{1}{3}\int_0^\infty v\,p\,n(p)\,\mathrm{d}p$
- $P = P_\mathrm{I} + P_\mathrm{e} + P_\mathrm{rad} = P_\mathrm{gas} + P_\mathrm{rad}$
  define $\beta = P_\mathrm{gas}/P \Rightarrow P_\mathrm{gas} = \beta P$, $P_\mathrm{rad} = (1-\beta)P$
- gas pressure
  $P_\mathrm{gas} = \mathcal{R}\rho\frac{T}{\mu}$
- degenerate electron pressure
  $$P_\mathrm{e,deg} = \frac{h^2}{20 m_\mathrm{e} \mathrm{u}^{5/3}}\left(\frac{3}{\pi}\right)^{2/3}\left(\frac{\rho}{\mu_\mathrm{e}}\right)^{5/3}$$
- relativistic degenerate electron pressure
  $$P_\mathrm{e,rel-deg} = \frac{hc}{8\mathrm{u}^{4/3}}\left(\frac{3}{\pi}\right)^{1/3}\left(\frac{\rho}{\mu_\mathrm{e}}\right)^{4/3}$$
- radiation pressure
  $P_\mathrm{rad} = \frac{1}{3}\int_0^\infty c\frac{h\nu}{c}\,n(\nu)\,\mathrm{d}\nu = \frac{1}{3}aT^4$

Regimes of the EOS



Different regimes of the equation of state as a function of T and ρ.

Electron Equation of State Regimes



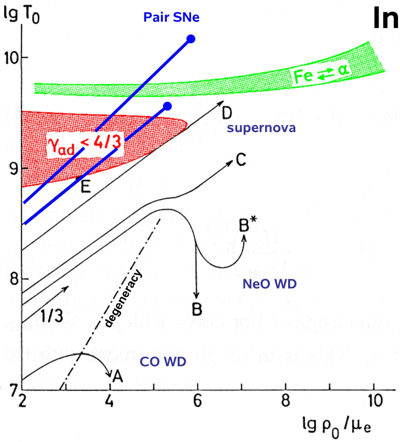
Electron-Positron Pair Production

- At $T \gtrsim 1\times10^9$ K photon can produce electron-positron pairs, from the highest energy photons of the Planck spectrum, $h\nu > 2m_\mathrm{e}c^2$:

$$e^+ + e^- \rightleftharpoons \gamma$$

- This converts radiation energy into rest mass of pairs
- hence compression increases pressure less
- $\Rightarrow$ adiabatic index γ_ad lower
- possible instability of star ($\gamma_\mathrm{ad} < \frac{4}{3}$)
  "pair instability supernova"
  ($\gamma_\mathrm{ad} \gtrsim \frac{4}{3}$ is needed for stability of stars, as we shall see later)

Electron-Positron Pair Production and Iron Dissociation



Kippenhahn & Weigert (1990)

Instability Regimes

adiabatic index < 4/3

Compression does not result in sufficient increase in pressure (gradient) to balance higher gravity at lower radius

e^+/e^--Pair Instability

Internal gas energy is converted into e^+/e^- rest mass (hard photons from tail of Planck spectrum)

Photo disintegration

Internal gas energy is used to unbind heavy nuclei into alpha particles and at higher temperature those into free nucleons

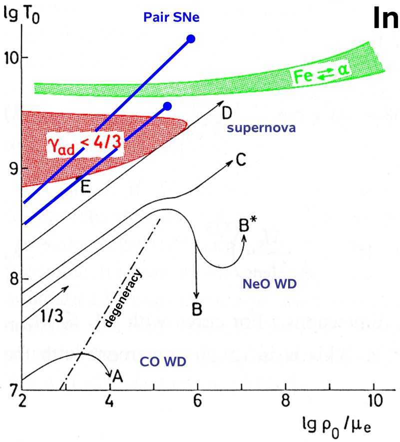
Iron Photo-Dissociation

- At very high temperatures in the stellar core, typically during the last stages of *massive* (and very massive) stars, including collapse of the iron core, iron can be dissociated, typically above $T > 7 \times 10^9$ K:

$$^{56}\mathrm{Fe} + \gamma \rightleftharpoons 13\,^{4}\mathrm{He} + 4\,\mathrm{n}$$

- This takes 100 MeV
- $\Rightarrow$ gas energy is used to unbind nucleus
- takes (about) as much energy as was released before to burn ^{4}He to ^{56}Fe
- $\Rightarrow \gamma_{\mathrm{ad}}$ drops
- $\Rightarrow$ possible instability of star (collapse)

Electron-Positron Pair Production and Iron Dissociation



Kippenhahn & Weigert (1990)

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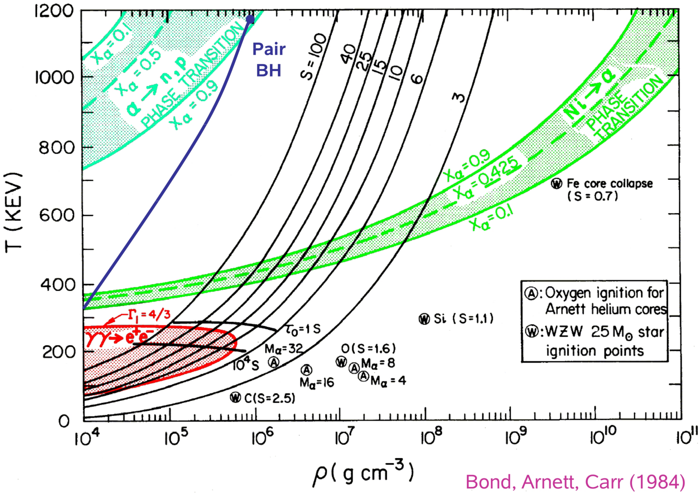
Helium Photo-Dissociation

- At even higher temperatures helium can be dissociated, typically above $T \gtrsim 10^{10}\,\mathrm{K}$:

$$^{4}\mathrm{He} + \gamma \rightleftharpoons 2\,\mathrm{n} + 2\,\mathrm{p}$$

- This takes $\sim 28\,\mathrm{MeV}$ *per* $^{4}\mathrm{He}$
- $\Rightarrow$ again, gas energy is used to unbind nucleus
- takes (about) as much energy as was released before to burn $4\,^{1}\mathrm{H}$ to $^{4}\mathrm{He}$
  (not counting neutrino losses during hydrogen burning)
- $\Rightarrow \gamma_{\mathrm{ad}}$ drops
- $\Rightarrow$ possible instability of star (collapse)

Helium and Iron Dissociation



Bond, Arnett, Carr (1984)

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2 Burning in Stars

Burning Phases in Stars

20 $M_\odot$ star

Fuel	Main Product	Secondary Product	T (10^9 K)	Time (yr)	Main Reaction
H	He	^{14}N	0.02	10^7	$4\,H \xrightarrow{CNO} {}^4He$
He	O, C	^{18}O, ^{22}Ne s-process	0.2	10^6	$3\,He^4 \rightarrow {}^{12}C$ $^{12}C(\alpha,\gamma)^{16}O$
C	Ne, Mg	Na	0.8	10^3	$^{12}C + {}^{12}C$
Ne	O, Mg	Al, P	1.5	3	$^{20}Ne(\gamma,\alpha)^{16}O$ $^{20}Ne(\alpha,\gamma)^{24}Mg$
O	Si, S	Cl, Ar, K, Ca	2.0	0.8	$^{16}O + {}^{16}O$
Si, S	Fe	Ti, V, Cr, Mn, Co, Ni	3.5	0.02	$^{28}Si(\gamma,\alpha)\ldots$